LITERATURE CITED

1. N. N. Zenger, Design of Pipelines for Permafrost Conditions (Experience at Noril'sk) [in Russian], Stroiizdat, Moscow (1964).
2. V. P. Stegantsev, Problems of the Far North [in Russian], No. 10, Izd. Akad. Nauk SSSR, Moscow (1964).
3. P. A. Bogoslovskii, Ice Conditions Occurring in Water-Station Pipelines [in Russian], Gosénergoizdat, Moscow – Leningrad (1950).
4. J. M. Savino and R. Siegel, Int. J. Heat Mass Transfer, 12, No. 7 (1969).
5. C. Lapadula and W. K. Mueller, Int. J. Heat Mass Transfer, 9, 702 (1966).
6. E. P. Martinez and R. T. Beabubouef, Canad. J. Chem. Eng., 50, No. 4 (1972).
7. M. N. Osisik and J. C. Mulligan, Trans. ASME, J. Heat Transfer, 91, Ser. C, No. 3 (1969).
8. N. Z. Gusev et al., Heat Calculations on Water Pipes Buried in Frozen Soil [in Russian], Izd. Irkutsk. Politekh. Inst., Irkutsk (1971).
9. S. S. Kutateladze and Yu. N. Krinchitskii, in: District-Heating Problems [in Russian], ONTI, Leningrad – Moscow (1936).
10. L. M. Al'tshuller, Zh. Tekh. Fiz., 29, No. 2 (1959).
11. N. Z. Frenkel', Hydraulics [in Russian], Gosénergoizdat, Moscow – Leningrad (1956).
12. B. A. Krasovitskii and F. S. Popov, Inzh. Fiz. Zh., 31, No. 2 (1976).
13. B. A. Krasovitskii and F. S. Popov, Inzh. Fiz. Zh., 29, No. 6 (1975).
14. L. S. Leibenzon, Handbook on Petroleum Mechanics, Part 1. Hydraulics [in Russian], GNTI, Moscow – Leningrad (1931).

ELECTRICAL SIMULATION OF THREE-DIMENSIONAL TEMPERATURE FIELDS OF ANISOTROPIC BODIES OF COMPLEX SHAPE

O. I. Bukovskaya, N. F. Dekhtyareva,
and L. A. Kozdoba

UDC 536.12:621.38

A procedure is described and results are presented of mathematical modeling of three-dimensional temperature fields on hybrid electrical combination simulators with integrated microcircuits.

The demands for more accurate thermal calculations increase every year. An increase in the reliability and an improvement of the quality of the elements are inseparably linked with optimization with respect to the thermal state in transient and steady-state regimes. These requirements force one to seek new methods and to solve two- and three-dimensional heat-conduction problems for bodies of complex shape with variable time- and temperature-dependent coefficients in both the basic equation and the boundary conditions of the mathematical model. For the most complex problems the only methods for investigating temperature distributions in three-dimensional structures involve numerical solutions on analog computers with a processor in the form of a network or a combination electrical model [1, 2]. These models are a subroutine of specialized hybrid computers permitting complete automation of the solution of field theory problems described by second-order partial differential equations [3].

In microminiature elements of electronic equipment similar to those shown in Fig. 1, three-dimensional temperature fields can be obtained only by mathematical modeling. The introduction of any temperature-measuring device leads to an inadmissible distortion of the temperature field, particularly inside the microelement. The thermal circuit of a hybrid integrated microcircuit is a typical example of a three-dimensional heat-conduction problem for an anisotropic body of complex shape. The schematic diagram does not show the leads. A special study with three-dimensional electrical models [4] showed that under certain conditions the effect of the leads on temperature fields can be neglected.

Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 35, No. 1, pp. 133-140, July, 1978. Original article submitted April 20, 1977.

0022-0841/78/3501-0854$07.50

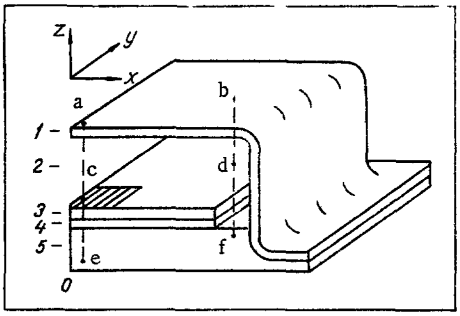

Fig. 1. Thermal circuit of hybrid integrated microcircuit: 1) cover; 2) air; 3) substrate; 4) adhesive; 5) flange; a-f) characteristic points on inner and outer surfaces.

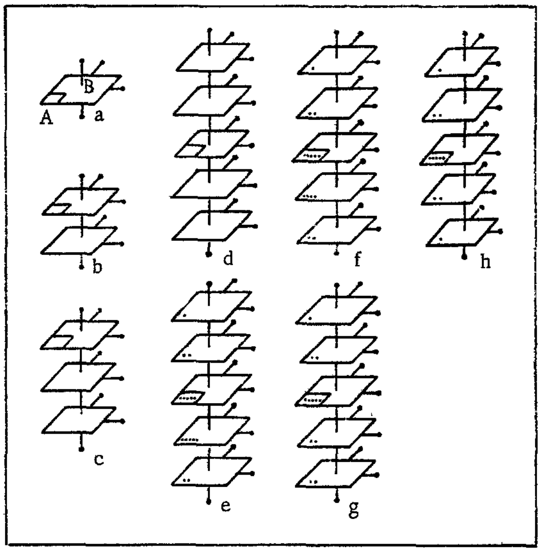

Fig. 2. Schematic diagrams of three-dimensional network and combination models. Discrete resistors (R_λ, R_α) along the z axis are not shown. The dots on the five-layer models are nodes in the heat-source region.

TABLE 1. Variations of Solutions on Various Types of Network and Combination Models.

Variant No.	Type of model	No. of planes	No. of nodes in a plane	$\alpha_u, \alpha_l, \alpha_s$
1	*R—R*	1	48	Different
2	*K—R*	1	19	»
3	*K—R*	1	19	Same
4	*K—R*	1	8	Different
5	*k—R*	1	8	»
6	*K*	2	8; 8	»
7	*K*	3	8; 8; 8	»
8	*K*	5	8; 8; 8; 8; 8	»

Note. K : k = 2 : 1. K and k are different scale combination models; R is the network resistance; the number of the variation is the number of the curve in Fig. 3.

The mathematical model of the thermal conditions of a three-dimensional anisotropic structure can be described by the following system of equations in rectangular coordinates:

$$\sum \frac{\partial}{\partial x_i}\left(\lambda_{ji}\,\frac{\partial T}{\partial x_i}\right) - c_j\rho_j\,\frac{\partial T}{\partial \tau} + q_{v_j} = 0, \qquad (1)$$

$$T(x_i,\ 0) = f(x_i), \qquad (2)$$

$$\alpha(T_a - T_s) = -\lambda_{ji}\frac{\partial T}{\partial n}, \tag{3}$$

$$\frac{\partial T}{\partial n} = 0. \tag{4}$$

A microcircuit is generally cooled from both sides. Therefore, condition (4) holds only on the planes of symmetry for the quarter of the circuit considered. On all the remaining surfaces, condition (3) holds. At the surfaces of contact of the circuit elements (Fig. 1), matching conditions are specified in the form of boundary conditions of ideal contact,

$$T_{s,j} = T_{s,j-1}; \quad -\lambda_{ji}\left.\frac{\partial T}{\partial n}\right|_s = -\lambda_{j-1,i}\left.\frac{\partial T}{\partial n}\right|_s. \tag{5}$$

Specifying ideal contact conditions is optional. The thermal contact resistance and the heat release at the contacting surfaces can be taken into account.

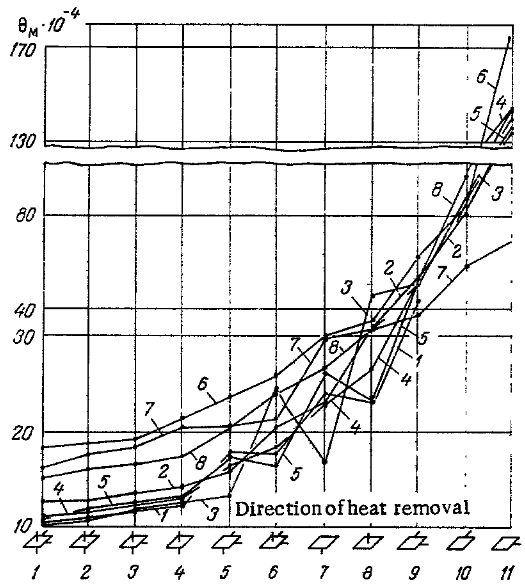

Fig. 3. The dependence of Θ_M (deg · cm²/W) on the direction of heat removal for various types of models and operating positions of the microcircuit. See Table 1 for explanation of numbers on curves.

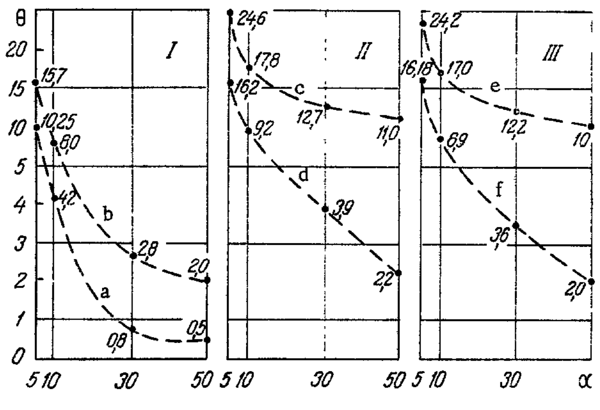

Fig. 4. Change of relative temperature Θ (deg · cm²/W) as a function of α (W/m³ · deg) at the center and on the periphery of various surfaces of a microcircuit ($\alpha_u = 1.3\alpha$, $\alpha_l = 0.7\alpha$, $\alpha_s = \alpha$); I) outer surface of cover; II) upper surface of substrate; III) lower surface of flange; a–f) correspond to points a–f of Fig. 1.

In our case α varied from 5 to 50,000 $W/m^2 \cdot {}^\circ K$, which enabled us to simulate cooling conditions from natural convection to semiconductive cooling (boundary conditions of the first kind). In order to take account of the direction of heat removal (from above, from below, or from the side) it was assumed that the microcircuit operates in the position shown in Fig. 1 with $\alpha_u = 1.3\alpha$, $\alpha_l = 0.7\alpha$, and $\alpha_s = \alpha$. In our case λ_{ji}, c_j, ρ_j, q_{vj}, and α are functions of the coordinates, the time, and the temperature; T_a is a function of coordinates and time. The layer of air between the cover and substrate is taken into account. The materials of the substrate, cover, flange, adhesive, and air layer have widely different thermophysical properties, differing by one to two orders of magnitude. The heat source is distributed uniformly over a given volume or according to the law $a_v(x_i, \tau)$. Special investigations with similar structures having relatively small thermal resistances across the thickness showed that the heat release can be simulated also by a surface heat source properly distributed over a given area S of the substrate. In this case the mathematical model (1)-(5) has boundary conditions of the second kind,

$$q = -\lambda_{ji} \frac{\partial T}{\partial n}. \tag{6}$$

The three-dimensional element in Fig. 1, which is one quarter of the microcircuit, consists of elementary plates – layers in which the thermal resistances along the vertical (z axis) are several times smaller than the thermal resistances in horizontal planes parallel to the xOy plane. In this case it is reasonable to construct a combination electrical model with electrically conducting paper and a network of ohmic resistors so that the planes of electrically conducting paper simulate the electrical (i.e., the corresponding thermal) resistances in horizontal planes. A plane of electrically conducting paper can replace a whole plate element of the microcircuit or simulate one of the elementary layers into which the cover, air layer, etc., are divided. Special methodical investigations showed that it is sufficient to take one plane (Fig. 1) for each element 1-5 of the microcircuit. Moreover, by taking account of the thermal resistance of the air layer along the z axis, it turns out that it is satisfactory to replace only the substrate, adhesive, and flange by such planes of electrically conducting paper.*

Figure 2 shows schematic diagrams of three-dimensional models having from one to five planes (a-h). We note that even the model which has only one plane is a three-dimensional model. In it, discrete wire-wound resistors are used to simulate thermal resistances for heat conduction along the z axis ($R_{\lambda z}$) and thermal resistances for external heat transfer ($R_{\alpha z}$) at the upper and lower outer surfaces of the microcircuit. In this case, just as in all other cases involving a combination of layers of materials with different λ, c, and ρ, the thermophysical properties simulated in such a three-dimensional model are the equivalent combined properties, different along different axes. These equivalent properties λ_{eq}, c_{eq}, and ρ_{eq} are calculated from relations given, for example, in [5]. Incidentally, it is appropriate to note that electrical models similar to those described in this article have been used successfully to determine λ_{eq}, particularly when the material consists of elements of irregular shape, and the coefficients λ of the elementary components are different in different directions, not only along different coordinate axes. In each plane (Fig. 2) the number of nodes was varied depending on the likely temperature gradients. The spatial intervals (elementary areas and volumes) were decreased as the heat source was approached. The number of nodes in a plane was varied from 8 to 49. The parameters of the combination model were calculated from expressions derived for asymmetric networks and combination models with nonuniform divisions [1]:

$$\begin{gathered} R_\lambda = \frac{l m_R}{\lambda_{ji} S}; \quad R_\tau = \frac{m_R \delta\tau}{c\rho v}; \quad R_\alpha = \frac{m_R}{\alpha S}; \\ R_{q_v} = \frac{(V_M - V_m)\, m_R}{q_v v m_V}; \quad R_q = \frac{(V_M - V_m)\, m_R}{q S m_V}. \end{gathered} \tag{7}$$

In Eqs. (7), l, S, and v are elementary lengths, areas, and volumes which are generally different for each node of the network. The procedure for solving field theory problems on asymmetric networks (networks with nonuniform divisions) is similar to the procedure used in the method of finite elements. In the finite-element method the conservation laws are generally written in variational form, but in electrical models with asymmetric divisions they are written in balance form. The parameters of network or combination models with capacitive and/or current-carrying elements were calculated from the following expressions:

$$\begin{gathered} I_{q_v} = q_v v m_V / m_R; \quad I_q = q S m_V / m_R; \\ C = c\rho v m_\tau / m_R; \quad m_\tau = R_e C / R_\tau c\rho v. \end{gathered} \tag{8}$$

---

*From now on we use the word planes to mean planes of electrically conducting paper.

The dots in Fig. 2e-h show the locations of the nodes in the various planes of the five-layer model in regions above and below the source. A similar division of a plane into elements and a procedure for connecting the nodes for unequal elementary areas in parallel planes permits a sharp decrease in the number of nodes in the model while maintaining a given accuracy. Figure 3 shows the maximum relative temperature $\Theta_M = (T_M - T_a)/q_V$ for one value of α as a function of the direction of heat removal, the type of model, the number of planes, and the number of nodes in a plane. A model consisting of a network of resistors is denoted by R–R. A model combining an R–R network (region A of Fig. 2a) and electrically conducting paper (region B of Fig. 2a) is denoted by K–R or k–R, depending on the scales of the combination models (k : K = 1 : 2). The data of Fig. 3 show that the results of the three-dimensional model with three planes are close to those of the model with five planes. In the three-layer model the values of R_λ are taken into account in all directions only in the substrate, adhesive, and flange; $R_{\lambda x}$ and $R_{\lambda y}$ are not taken into account in air and the cover. In the five-layer model R_λ is taken into account for air and the cover in the horizontal plane. A comparison of data for the three- and five-layer models indicates that because of the small thermal conductivity of air one could take account only of $R_{\lambda z}$ for air and $R_{\lambda z}$ for the cover. Of course, the value used for the thermal conductivity of air took account of convection in the thin layer.

As noted above, the values used for α_u, α_l, and α_s depended on the operating position of the microcircuit: $\alpha_u = 1.3\alpha$, $\alpha_l = 0.7\alpha$, and $\alpha_s = \alpha$. Figure 3 shows that the value of Θ_M depends on the direction of heat removal (in all directions, upward, downward, toward the side, etc.), the type of model, the number of planes, the number of nodes in a plane, and whether the values of α are identical or depend on the operating position of the microcircuit. Special investigations showed that when α is varied from 5 to 50,000 $W/m^2 \cdot {}^\circ K$, the conclusions about the optimum direction of heat removal are changed because of the change of the ratios of R_λ in various directions and R_α. Mathematical modeling of three-dimensional temperature fields on electrical simulators enables one not only to obtain interesting heat-engineering results, but also to optimize with respect to accuracy, solution time, cost of solution, and finite-difference models of Eqs. (1)-(6) when all-purpose digital computers are used to solve the equations. Models which give maximum accuracy for a minimum number of nodes are defined as optimal. The choice of accuracy depends on the particular thermal problem. It is quite clear that to optimize a product or its operating conditions the accuracy of the calculation must be much higher (an error of no more than ± 1%) than in sketchy designs (admissible error ± 10-25%).

Figure 3 shows also that complicating the model by increasing the number of planes or the number of nodes in a plane does not necessarily affect the maximum temperature. Analysis of three-dimensional fields by a microcircuit showed that this indeterminacy arises from the anisotropy of properties and from the variety of ratios of internal and external thermal resistances which is characteristic of such products. Particular attention must be paid to the choice of the thermal circuit for variations of α and q and changes in the directions of heat removal. In a number of cases for heat removal from one side, not only two-dimensional, but also one-dimensional heat-conduction problems can be studied on the mathematical model.

A comparison of curves 2 and 3 of Fig. 3 shows that Θ_M is affected by the transition from constant values of α on the outer cooled surfaces to different values of α, taking account of the operating position of the microcircuit ($\alpha_u = 1.3\alpha$, $\alpha_l = 0.7\alpha$, and $\alpha_s = \alpha$). Of course, the magnitude of this effect depends on the direction of heat removal. Study of three-dimensional temperature fields has established that for cooling from one side both the choice of the direction of heat removal (it is desirable to cool the side with minimum R_λ) and the value of α are particularly important. For values of α which are characteristic for natural convection, a change in α by 10-30% has a definite effect on Θ_M.

It is typical that an increase in the number of planes and nodes does not ensure an increase in the accuracy of the solution. For example, a model with two planes (curve 6 of Fig. 3) gives values of Θ_M lying between the values for the one- and five-layer models. Such "paradoxes" occur rather frequently in numerical studies of complex heat-conduction problems [6]. In the present case this paradox arises from the change in the ratio of the thermal resistances when the anisotropic volume is divided into 1, 2, 3, and 5 elementary layers. It can be seen from Fig. 3 that the changes in Θ_M are substantial and depend on the direction of heat removal. Changes in the direction of heat removal cause radical changes in thermal resistances from the source to the cooling surfaces.

The removal of heat in only one direction (especially upwards or laterally) leads to a catastrophic increase in Θ_M. This results not only in a sharp decrease in the reliability of the element [7], but can lead to prompt breakdown even when operating at powers 5-10% of nominal.

The determination of three-dimensional temperature fields on combination electrical models enables one to estimate the temperature drops across the outer surfaces of a microcircuit whether they are cooled or not. Figure 4 shows temperature changes at certain characteristic internal points in the region of the heat source, on the substrate, and on the upper and lower surfaces of a microcircuit. It is clear that the temperatures on the outer surfaces vary considerably from the center to the periphery. These drops make a substantial change in the so-called thermal resistance of the microcircuit. The thermal resistance of the whole microcircuit, determined experimentally or calculated by assuming equality of the temperatures of the outer surfaces of the elements of electronic equipment, are considerably different from the actual values. This difference increases with increasing heat-transfer coefficients (cf. Fig. 4). Figure 4 shows the effect of α on Θ for cooling from all sides. Similar data are obtained in other variations of heat removal. Optimization of the elements with respect to temperatures and optimization of operating conditions must be performed by starting from the three-dimensional temperature distributions. Only in this way is it possible to achieve a sharp increase in the reliability of apparatus, which changes substantially with each degree rise in the maximum temperature. According to statistical data [7], for each 10° rise in temperature the reliability on the average decreases by 25%. Therefore, electrical simulators permitting highly accurate investigations of three-dimensional temperature fields under various conditions are indispensible in the design of elements of electronic apparatus.

NOTATION

T, temperature; V, voltage; C, capacitance; R, resistance; I, current; $m_R = R_e/R_T$, $m_V = (V_{max} - V_{min})/(T_{max} - T_{min})$, $m_\tau = \tau_e/\tau_T$, scales of electrical and thermal quantities: resistances, potentials, and times; $i = 1, 2, 3$; j, number of different components; m, number of node. Indices: u, upper; l, lower; s, side; T, thermal; e, electrical; M, maximum; s, surface; a, ambient.

LITERATURE CITED

1. L. A. Kozdoba, Electrical Simulation of Heat- and Mass-Transfer Phenomena [in Russian], Énergiya, Moscow (1972).
2. L. A. Kozdoba, Vopr. Radioélektron., Ser. TRTO, No. 1, 61 (1974).
3. Machine Methods for Solving Engineering Problems in Field Theory [in Russian], Naukova Dumka, Kiev (1976).
4. O. I. Bukovskaya and L. A. Kozdoba, Vopr. Radioélektron., Ser. TRTO, No. 3, 99 (1975).
5. G. N. Dul'nev and Yu. P. Zarichnyak, Thermal Conductivity of Mixtures and Composite Materials [in Russian], Énergiya, Leningrad (1974).
6. L. A. Kozdoba, Solutions of Nonlinear Heat-Conduction Problems [in Russian], Naukova Dumka, Kiev (1976).
7. L. L. Rotkop and Yu. E. Spokoinyi, Guarantee of Thermal Conditions in the Design of Electronic Equipment [in Russian], Sov. Radio, Moscow (1976).